Vibration of Plates Considering Shear and Rotatory Inertia

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Introduction

THE large-amplitude vibrations of plates and shells have been receiving considerable attention in the literature in recent years. In particular, rectangular plates have received the attention of many investigators. The results reported so far do not, in many cases, seem to incorporate the effects of transverse shear and rotatory inertia. Wu and Vinson[1] investigated the effects of transverse shear and rotatory inertia on the large-amplitude vibration of isotropic, transversely isotropic, and orthotropic rectangular plates, based on assumed expressions not only for the mode shape w but also for the rotation components α and β. The purpose here is to suggest an improvement to the solutions given by Wu and Vinson by altogether eliminating α and β from the governing equations, thereby avoiding the necessity to proceed on assumed approximations for α and β. The equations developed here are based on Berger approximation. Based on Galerkin's method and an assumption of only mode shape w, the effects of geometric nonlinearity, transverse shear deformation, and rotatory inertia are reported for certain cases of isotropic and orthotropic rectangular plates clamped along the boundaries.

Analysis

The governing equations for the large-amplitude free undamped vibration of rectangular orthotropic plates wherein transverse shear and rotatory inertia effects are considered are[2]

$$e=\epsilon_1+\lambda_1\epsilon_2=\delta^2h^2/12 \tag{1}$$

$$\alpha+w_{,x}-K_1\alpha_{,xx}-K_2\alpha_{,yy}-K_3\beta_{,xy}+K_4\alpha_{,tt}=0 \tag{2}$$

$$\beta+w_{,y}-K_5\beta_{,yy}-K_6\beta_{,xx}-K_7\alpha_{,xy}+K_8\beta_{,tt}=0 \tag{3}$$

$$I-K_9(\alpha_{,x}+w_{,xx})-K_{10}(\beta_{,y}+w_{,yy})-\rho h w_{,tt}=0 \tag{4}$$

where

$$I=C_1e(w_{,xx}+\lambda_1 w_{,yy}),\ C_1=E_1h,\ \lambda_1=(E_2/E_1)^{1/2}$$

$$E_1=E_\xi/\nu',\ E_2=E_\eta/\nu',\ K_1=K_{11}E_1,\ K_2=GK_{11}$$

$$K_3=G\,E_2K_{11}K_{13},\ K_4=\rho K_{11},\ K_5=K_{12}E_2$$

$$K_6=G\,K_{12},\ K_7=G\,E_2K_{12}K_{13},\ K_8=\rho K_{12}$$

$$K_9=-(5hG_2)/6,\ K_{10}=-(5hG_3)/6,\ K_{11}=h^2/(10G_2)$$

$$K_{12}=h^2/(10G_3),\ K_{13}=(1/E_2+\nu_{\eta\xi}/G)$$

$$\nu'=(1-\nu_{\eta\xi}\nu_{\xi\eta}),\ \epsilon_1=u_{,x}+{1/2}w_{,x}^2,\ \epsilon_2=v_{,y}+{1/2}w_{,y}^2$$

u, v, w are the displacement components of the plate, h the thickness, ρ the mass per unit volume of the plate, α and β the rotation components in the x and y directions, respectively, of the plate, and E_ξ, E_η, $\nu_{\xi\eta}$, $\nu_{\eta\xi}$, G, G_2, G_3 are the elastic constants of the orthotropic plate material. Since Eqs. (1-4) are coupled and nonlinear, exact solutions to these are difficult, and, hence, an attempt is made here to obtain approximate solutions.

Solving for α and β from Eqs. (2) and (3) and substituting for these in Eq. (4), we get

$$N(I_1)+R(w)=0 \tag{5}$$

where the operators N and R are defined as

$$N=m_1\frac{\partial^4}{\partial x^4}+m_2\frac{\partial^4}{\partial x^2\partial y^2}+m_3\frac{\partial^4}{\partial y^4}+m_4\frac{\partial^4}{\partial t^4}+m_5\frac{\partial^4}{\partial x^2\partial t^2}$$
$$+m_6\frac{\partial^4}{\partial y^2\partial t^2}+m_7\frac{\partial^2}{\partial x^2}+m_8\frac{\partial^2}{\partial y^2}-m_9\frac{\partial^2}{\partial t^2}-1$$

$$R=m_{10}\frac{\partial^4}{\partial x^4}+m_{11}\frac{\partial^4}{\partial y^4}+m_{12}\frac{\partial^4}{\partial x^2\partial y^2}-K_9\frac{\partial^2}{\partial x^2}-K_{10}\frac{\partial^2}{\partial y^2}$$
$$+m_{13}\frac{\partial^4}{\partial x^2\partial t^2}+m_{14}\frac{\partial^4}{\partial y^2\partial t^2}$$

$$m_1=-K_1K_6,\ m_2=K_3K_7-K_1K_5-K_2K_6,\ m_3=-K_2K_5$$

$$m_4=-K_4K_8,\ m_5=K_1K_8+K_4K_6,\ m_6=K_2K_8+K_4K_5$$

$$m_7=K_1+K_6,\ m_8=K_2+K_5,\ m_9=K_4+K_8,\ m_{10}=K_6K_9$$

$$m_{11}=K_2K_{10},\ m_{12}=K_9(K_5-K_3)+K_{10}(K_1-K_7)$$

$$m_{13}=-K_8K_9,\ m_{14}=-K_4K_{10}$$

$$I_1=I-K_9w_{,xx}-K_{10}w_{,yy}-\rho h w_{,tt}$$

Equations (1) and (5) are then the equations to be solved in order to study the effects of transverse shear and rotatory inertia on the large-amplitude vibration of orthotropic rectangular plates.

Numerical Example

Considering a rectangular plate of dimensions 2a, 2b clamped along the boundaries, a mode shape for w is assumed to satisfy all the boundary conditions as

$$\frac{w}{h}=\frac{f(\tau)}{4}\left(1+\cos\frac{\pi x}{a}\right)\left(1+\cos\frac{\pi y}{b}\right) \tag{6}$$

Assuming that the edges of the plate are immovable and substituting for ϵ_1 and ϵ_2 in terms of the displacements u, v, and w and integrating both sides of Eq. (1), δ^2 is obtained as

$$e=\frac{\delta^2h^2}{12}=-\frac{1}{8ab}\int_x\int_y w(w_{,xx}+\lambda_1 w_{,yy})\,\mathrm{d}x\mathrm{d}y \tag{7}$$

For an assumed w as given by Eq. (6), e is computed from Eq. (7). With this expression for e, the Galerkin's method is applied on Eq. (5) to arrive at a model equation of the following form

$$n_1f+n_2\ddot{f}+n_3\ddddot{f}+n_4\overset{\cdot\cdot}{\ddddot{f}}+n_5f(\dot{f})^2+n_6\ddot{f}f^2+n_7f^3$$
$$+n_8f\dot{f}\dddot{f}+n_9f(\ddot{f})^2+n_{10}\ddot{f}(\dot{f})^2+n_{11}\ddddot{f}f^2=0 \tag{8}$$

where

$$n_1=a_{30},\ n_2=a_{31}q,\ n_3=a_{32}q^2,\ n_4=a_{23}q^3$$

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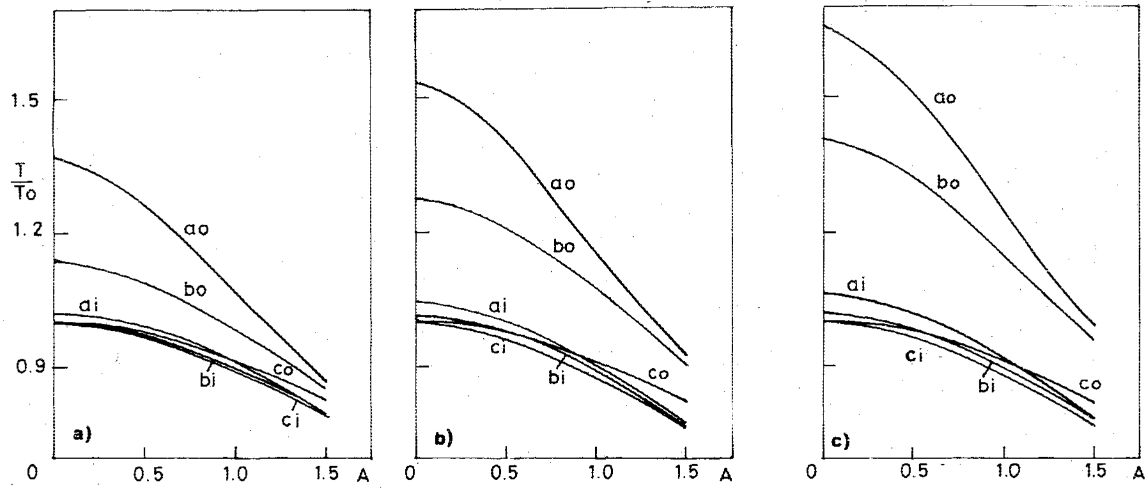

Fig. 1a) T/T_0 vs amplitude, $r=0.5$; b) T/T_0 vs amplitude, $r=1.0$; and c) T/T_0 vs amplitude, $r=1.5$. *a*, $h/2a=1/10$; *b*, $h/2a=1/20$; *c*, no shear or rotatory inertia effects; *o*, orthotropic; and *i*, isotropic.

$$n_5=2(a_{17}+a_{19})q,\ n_6=n_5/2,\ n_7=a_{16},\ n_8=8a_{18}q^2$$

$$n_9=\frac{3a_8}{4},\ n_{10}=\frac{3n_8}{2},\ n_{11}=\frac{n_8}{8},\ a_1=\frac{3h^2ma_4}{128}$$

$$a_2=-\frac{E_1h^2a_1m}{4},\ a_3=m_1m^2-m_7m-1,\ a_4=1+\lambda_1r^2$$

$$a_5=m_3n^2-m_8n,\ a_6=m_2mn,\ a_7=a_8=a_{11}=0$$

$$a_9=m_5m,\ a_{10}=m_6n,\ a_{12}=K_9+K_{10}r^2$$

$$a_{13}=a_3+a_5+a_6,\ a_{14}=3m(K_9+m_{10}m)+3n(K_{10}+m_{11}n)$$

$$+m_{12}mn,\ a_{15}=-(m_{13}m+m_{14}n),\ a_{16}=a_2\{2a_3+a_4a_{13}$$

$$+2\lambda_1r^2(a_5-1)\},\ a_{17}=-9m_9a_2a_4,\ a_{18}=9m_4a_2a_4$$

$$a_{19}=-3a_2\{2a_9+a_4(a_9+a_{10})+2\lambda_1r^2a_{10}\}$$

$$a_{20}=(mh/4)\{a_{12}a_{13}+2a_3K_9+2K_{10}r^2(a_5-1)\}$$

$$a_{21}=-p\{-4+2a_3+2(a_5-1)+a_{13}\},\ a_{22}=9pm_9$$

$$a_{23}=-9pm_4,\ a_{24}=-3mha_{12}m_9/4$$

$$a_{25}=(-mh/4)\{2K_9a_9+2K_{10}r^2a_{10}+(a_9+a_{10})a_{12}\}$$

$$a_{26}=\frac{3mhm_4a_{12}}{4},\ a_{27}=\frac{ha_{14}}{4},\ a_{28}=\frac{3ha_{15}}{4}$$

$$a_{29}=3p(a_9+a_{10}),\ a_{30}=a_{20}+a_{27},\ a_{31}=a_{21}+a_{24}$$

$$+a_{25}+a_{28},\ a_{32}=a_{22}+a_{26}+a_{29},\ m=\frac{\pi^2}{a^2},\ n=\frac{\pi^2}{b^2}$$

$$p=\frac{\rho h^2}{4},\ q=\frac{E_\xi}{\rho a^2},\ r=\frac{a}{b},\ (\dot{\ })=\frac{d}{d\tau},\ \tau=tq^{1/2}$$

The modal Eq. (8) is solved using the numerical Runge-Kutta method. The period-amplitude relationship applicable for the large-amplitude vibration of orthotropic and isotropic rectangular plates is plotted in Fig. 1 where T/T_0 is the ratio of the nonlinear period of vibration, including the effects of transverse shear and rotatory inertia, to the corresponding linear period for a classical plate not including these effects and $r(=a/b)$ is the aspect ratio of the plate. The material constants of the plate are assumed to be the same as in Ref. 1.

Results

The effects of transverse shear deformation and rotatory inertia on the large-amplitude vibration of rectangular plates are shown by an increase in the nonlinear period although the increase is less at moderately large amplitudes. As can be seen from Fig. 1, the period-amplitude relationship is of the hardening type. The effect of the particular type of orthotropy that is considered here is to produce a significant increase in the nonlinear period, the increase being more at high aspect ratios. It is thus seen that the effects of transverse shear deformation and rotatory inertia play a considerably important role in the large-amplitude vibration of orthotropic plates.

References

[1] Cheng, Ih Wu and Vinson, J. R., "On the Nonlinear Oscillations of Plates Composed of Composite Materials," *Journal of Composite Materials*, Vol. 3, July 1969, pp. 548-561.

[2] Sathyamoorthy, M., "Shear and Rotatory Inertia Effects on Large Amplitude Vibration of Skew Plates," *Journal of Sound and Vibration*, Vol. 52, May 1977, pp. 155-163.